ISO, MPC and PID: The Good, The Bad and The Ugly Discrete Controllers

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Abstract

In this paper, three well-known discrete controllers - Infinite Steps Optimal (ISO), Model Predictive Control (MPC) and Proportional Integral Derivative (PID) - are discussed and compared. The ISO controller is the perfect controller; it should be the industry standard. The MPC controller is a heterodox controller, which should be scrutinized before application. The PID controller is acceptable for low-order control systems - a fact known for a long time by control engineers.

Keywords: $\mathbf{H}_\infty$ control, Kalman filtering, model predictive controller, PID controller.

1 Introduction

The movie *The Good, The Bad and The Ugly* is an American movie with three western movie icons: Clint Eastwood (The Good), Lee van Cleef (The Bad) and Eli Wallach (The Ugly), directed by Sergio Leone. It is a movie about the American Civil War, but it is also a movie about moral and religious teaching: The Bad must be dead. While it is only entertainment per se, the movie really describes this type of structure: Good, Bad and Ugly in many aspects of our human society. Take the control system engineering for example, we see The Good (controller), The Bad (controller) and The Ugly (controller).

The Good controller is the ISO controller, developed to perfection by AuLac Technologies Inc.. The Bad controller is the MPC controller. The Ugly controller is the well-known PID controller. In this paper, we discuss the nomination for the controllers and the controllers themselves. The paper is organized as follows. Section one is the introduction section. Section two describes the ISO controller. In section three, the MPC controller is presented. In section four, the PID controller is discussed and section five concludes the discussion of the paper.

2 The ISO Controller

Since the MPC controller is usually discussed in a stochastic environment, we will restrict our discussion to the problem of stochastic regulating control for a fair discussion. The block diagram for this feedback control system is described by the following Fig. 1.

From Fig. 1, we can write

$$\begin{aligned} y_t &= G_p(z^{-1})u_t + G_d(z^{-1})a_t, \\ &= \frac{\omega(z^{-1})}{\delta(z^{-1})}u_{t-f-1} + \frac{\theta(z^{-1})}{\phi(z^{-1})}a_t. \end{aligned} \quad (1)$$

The above equation is known as the Box-Jenkins model. Reference G. Box and G. Jenkins (1976) gives a fundamental discussion of this model. In reference K. Vu (2008), the author lists two control criteria for this model:

$$\underset{u_t}{Min} \quad E\{y_{t+f+1}^2 + \lambda u_t^2 \mid \mathcal{Y}_t\} \quad (2)$$

and

$$Min\ \sigma_y^2 + \lambda\sigma_u^2$$

where σ_y^2 and σ_u^2 are the variances of the output and input variables under feedback.

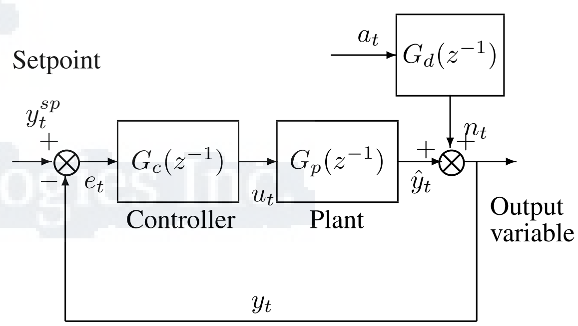

Figure 1. A Regulating Feedback Control System.

The first control criterion gives the one step optimal controller. The one step optimal controller sees and controls only one point at a time. It is, therefore, short-sighted and earns the name as the *myopic controller*. It is a controller that is vulnerable to nonminimum phase systems. In the equivalent state space model, the one step optimal controller gives the separation theorem with Kalman filtering. The second control criterion gives the infinite steps optimal controller because the variance formula of the input variable σ_u^2, given in K. Vu (2007), involves an infinite number of control actions from the input variable. Reference K. Vu (2008) gives the following equations that lead to the procurement of this controller, viz

$$\begin{aligned} \alpha(z)\alpha(z^{-1}) &= \omega(z)\omega(z^{-1}) + \lambda\delta(z)\delta(z^{-1}), \\ \frac{\theta(z^{-1})}{\phi(z^{-1})} &= \psi(z^{-1}) + \frac{\gamma(z^{-1})}{\phi(z^{-1})}z^{-f-1}, \end{aligned}$$

$$\frac{\gamma(z^{-1})\omega(z)}{\phi(z^{-1})\alpha(z)} = \frac{\beta(z^{-1})}{\phi(z^{-1})} + \frac{\zeta(z)}{\alpha(z)} z.$$

The optimal controller can be obtained as

$$u_t = -\frac{\delta(z^{-1})\beta(z^{-1})}{\alpha(z^{-1})\theta(z^{-1}) - \omega(z^{-1})\beta(z^{-1})z^{-f-1}} y_t,$$

and it gives the minimal value of the control criterion or performance index as

$$\hat{\sigma}^2 = \underset{z=0}{Residue}\Big[\psi(z)\psi(z^{-1})\frac{1}{z} + \lambda\frac{\delta(z)\gamma(z)\delta(z^{-1})\gamma(z^{-1})}{z\alpha(z)\phi(z)\alpha(z^{-1})\phi(z^{-1})} + \frac{\zeta(z)\zeta(z^{-1})}{z\alpha(z)\alpha(z^{-1})}\Big].$$

This is a research result of AuLac Technologies Inc.. The infinite steps optimal control algorithm for the ARMAX model has been obtained by other researchers. The algorithms for this model or slightly different models and the AuLac Technologies Inc.'s algorithm for this model are described in K. Vu (2008a).

While the aforementioned controller is infinite steps optimal, the fact that the whole spectrum is shrunk to a single value as the variance for optimization really limits the infinity of the controller. The $\mathbf{H}_\infty$ controller described in K. Vu (2008b), which can retain much of the infinity nature of the controller by having some relative freedom of shaping the output variable spectrum with a choice of a frequency weighting function, is a controller with much more infinity nature.

Now if we can imagine that we take the block with the transfer function $G_d(z^{-1})$ in Fig. 1 and slide it to the position in front of the setpoint y_t^{sp}, then we have a tracking control problem. Because the problem is now deterministic, the white noise a_t in this case will change to the Dirac delta sequence δ_t and the variances will yield their places to the sums of squares. The infinite steps optimal tracking controller will be

$$u_t = \frac{\delta(z^{-1})\beta(z^{-1})}{\alpha(z^{-1})\theta(z^{-1}) - \omega(z^{-1})\beta(z^{-1})z^{-f-1}} e_t.$$

This infinite steps optimal tracking controller has only one degree of freedom (1-DOF). It can be designed for a nonminimum-phase control system like the Vogel-Edgar or IMC controller. But all of these controllers cannot prevent an inverse response of a nonminimum-phase control system. Only the 2.5-DOF controller described in K. Vu (2008), a research result of AuLac Technologies Inc., can prevent this inverse response because it has an additional path (DOF) with the future setpoint values.

With the $\mathbf{H}_\infty$ regulating controller and the 2.5-DOF tracking controller, the ISO controller really deserves the nomination of The Good controller. It is the modern sage Albert Einstein who said: *"Two things are infinite: the universe and human stupidity; and I'm not sure about the universe."* As the control engineer who has put the final touch to these infinite steps optimal controllers, the author of this paper wants to modify this saying to the following one: *"Human wisdom is infinite; it is the infinite wisdom of many control researchers that gives the infinite steps optimal (ISO) controllers."'*

3 The MPC Controller

In Eq. (2), if we have a sum of observations rather than a single observation, then the performance index will become

$$\underset{u_t}{Min}\ E\{\sum_{t=t_0}^{N} y_{t+f+1}^2 + \lambda u_t^2 \mid \mathcal{Y}_t\},$$

and we have the criterion of the finite N steps optimal control algorithm. In the above equation, we see that u_t affects y_{t+f+1}. But the dynamics of the control system carries to the next observation and that means u_t also affects many observations of the output variable after the time $t+f+1$. The sum means that the optimization involves more control actions than just one given by u_t. The condition, $\mathcal{Y}_t$, of the expectation operator E, however, prevents the calculation of u_{t+1} and other control actions in the future of t because legally the time $t+1$ has not occurred yet.

The philosophy, controversy and idiosyncrasy of predictive control is that it allows the future control actions of the input variable to be calculated. To legalize the illusion, the architects of model predictive control remove the condition in the performance index without knowing that the expected quantity will then become an unconditional expectation, which will give the variance of a time series not the square of a variable. As a result of this removal, the sum now must be a sum of weighted variances of the output and input variables. In references D. Clarke, C. Mohtadi and P. Tuffs (1987) and D. Clarke, C. Mohtadi and P. Tuffs (1987), the performance index of a generalized predictive controller is

$$Min\ E\{J(N_1, N_2)\} = Min\ E\{\sum_{j=N_1}^{N_2} [\psi_{t+j} - w_{t+j}]^2 + \sum_{j=1}^{N_2} \lambda(j)[\Delta u_{t+j-1}]^2\}$$

where

ψ_t	is $P(z^{-1})y_t$,
N_1	is the minimum costing horizon,
N_2	is the maximum costing horizon,
$\lambda(j)$	is a control-weighting sequence

for the control model

$$A(z^{-1})y_t = B(z^{-1})u_{t-1} + \frac{C(z^{-1})}{\Delta}\varepsilon_t.$$

In D. Clarke, C. Mohtadi and P. Tuffs (1987), the future control actions are specified as

$$\Delta u_{t+j-1} = 0, \qquad j > NU.$$

The parameter NU is called the control horizon.

While predictive control can vary a little bit from algorithm to algorithm, the pivotal point in this control strategy is the future control actions. Will the algorithm calculate a number of successive control actions of the input variable and implement them, then only repeat this endeavor when the number of calculated control actions is depleted or change its mind by recalculating some control actions and ignoring what it just did the last time? There are issues with predictive control if it is applied to stochastic regulating control where it is usually marketed.

Model predictive control, however, can be applied to deterministic tracking control where prediction does not cost an arm or a leg and where it can be done legally. In this application, model predictive control provides an easy way to shape the input variable response. In tracking control, a control engineer can calculate an infinite number of control actions when a setpoint-change model is given. Tracking control, in effect, does not need the feedback signal. The feedback signal is used only as an added assurance of the accuracy of the transfer function model.

The MPC controller is nominated as The Bad controller because it does what is contrary to common belief or practice: The future control actions must be calculated and implemented in the future.

4 The PID Controller

In Eq. (1), if the polynomials and parameter of the control system are:

$$\begin{aligned}\omega(z^{-1}) &= \omega_0,\\ \delta(z^{-1}) &= 1-\delta_1 z^{-1}-\delta_2 z^{-2},\\ \theta(z^{-1}) &= 1-\theta_1 z^{-1},\\ \phi(z^{-1}) &= 1-z^{-1},\\ f &= 0,\end{aligned}$$

then the minimum variance controller for the system will be a PID controller with the following gains:

$$k_p = -\frac{(1-\theta_1)(\delta_1+2\delta_2)}{\omega_0}, \quad (3)$$

$$k_i = -\frac{(1-\theta_1)(1-\delta_1-\delta_2)}{\omega_0}, \quad (4)$$

$$k_d = \frac{(1-\theta_1)\delta_2}{\omega_0}. \quad (5)$$

The PID controller is then the optimal controller for low-order systems with no dead time and transmission zeros. The controller has the following form:

$$\begin{aligned}u_t &= k_p y_t + k_i \sum_{l=1}^{t} y_l + k_d(y_t - y_{t-1}),\\ &= k_p y_t + k_i \frac{1}{1-z^{-1}} y_t + k_d(1-z^{-1}) y_t,\end{aligned}$$

where k_p is the proportional gain, k_i is the integral gain and k_d is the derivative gain. A number of authors alter the above equation of the controller. The alteration is usually in the integral and derivative terms. Any alteration in either term will bring catastrophic effect to the performance of the controller. The reason for this fact is, as explained in K. Vu (2008), the relation of the three gains must reflect the relation of open loop and feedback controls.

From the last equation, we can write

$$(1-z^{-1})u_t = (k_p+k_i+k_d)y_t - (k_p+2k_d)y_{t-1} + k_d y_{t-2}.$$

The above equation is called the velocity form of the controller. It is a more convenient form than the positional form described by an earlier equation. The form prevents integral windup, and the input variable u_t can either be a deviation variable or a full-value variable. In this form, we can see very clearly that the controller does not remember the past values of the input variable. For a control system with a dead time ($f \neq 0$), its controller must remember the past values of the input variable. The PID controller, therefore, is vulnerable to control systems with a dead time.

To find the physical effect of the gains on the performance of the control loop, we set $\theta_1 = 0$ in Eqs. (3), (4) and (5). By setting δ_2 in Eq. (5) to zero, we can say that the derivative gain is used for an extension to second-order transfer functions. Similarly by setting $\delta_1 = \delta_2 = 0$ in Eq. (3), we can tell that the proportional gain is used to boost a sluggish loop for a faster response. Eq. (4) tells us that the integral gain should be equal to the inverse of the open-loop gain of the transfer function.

With all the characteristics of the PID controller presented, we can say that the PID controller is an acceptable controller. It gives the performance of an optimal controller when the orders of the control system are low. As a result, it is usually seen as the controller to control machines and devices. It is, however, called The Ugly controller because it refuses to adapt and change its form for better performance.

5 Conclusion

In this paper, we have discussed three popular discrete controllers: the ISO, the MPC and the PID. The ISO controller should be the standard controller in the control industry for its strength. While it is ugly, the PID controller has had a long life and it could exist forever. The MPC controller is a heterodox controller. In the movie *The Good, The Bad and The Ugly*, we see The Bad is ended by The Good. With a major defect, whether the MPC controller stands the test of time or it will quickly fade into oblivion and become a second class control technology, only time can tell. For control engineers, nonetheless, it is time to reassess their control algorithms and fine tune them to perfection.

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